Morse-Smale Complexes for Piecewise Linear 3-Manifolds

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Joint work with

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Problem

Given Morse function $f : M^3 \rightarrow R$

Morse-Smale complex is a topological structure partitioning space into regions of unifom gradient flow

Define Morse-Smale complexes and give an algorithm to construct them for piecewise linear data

Motivation

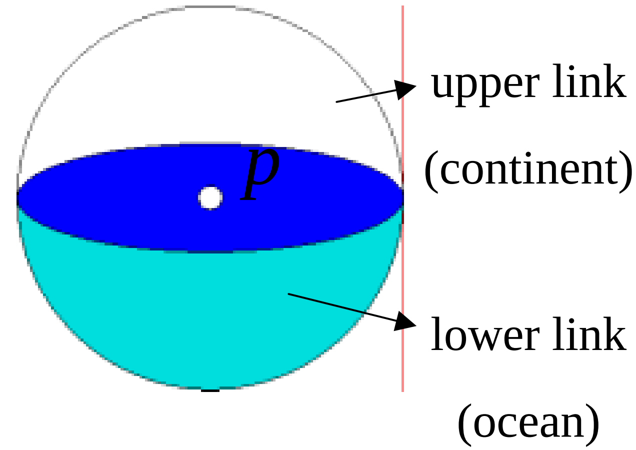
Morse function $f : M^3 \rightarrow \mathbb{R}$

X-ray crystallography	electron density
MRI	proton density
Lattice dislocation	atom density

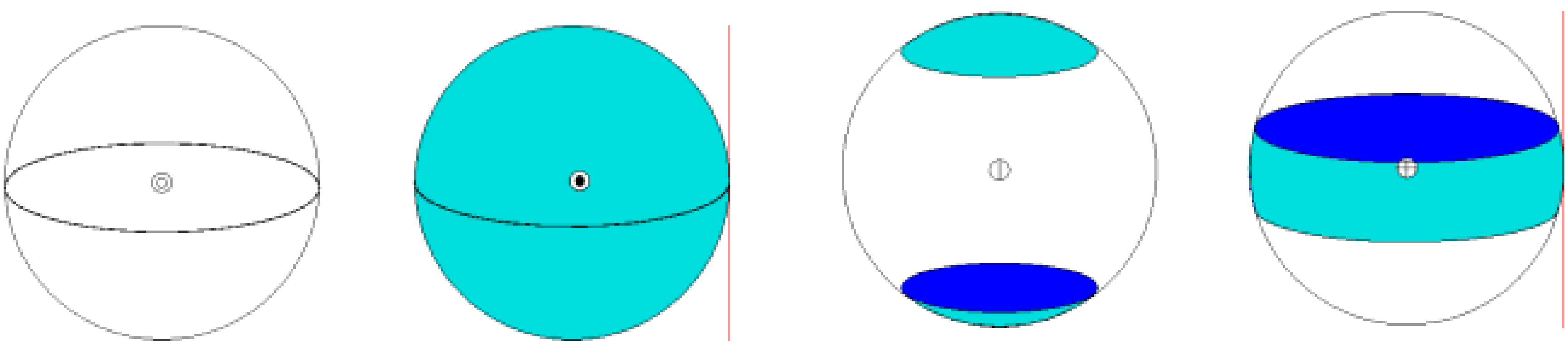
Critical Points

- Have zero gradient
- Characterized by lower link

regular



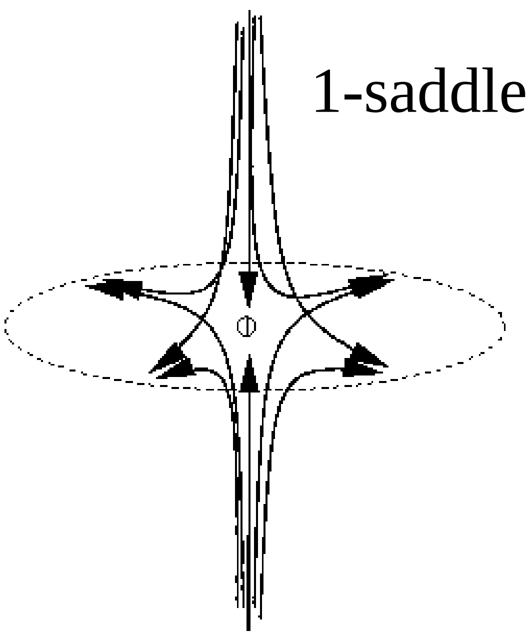
minimum maximum 1-saddle 2-saddle

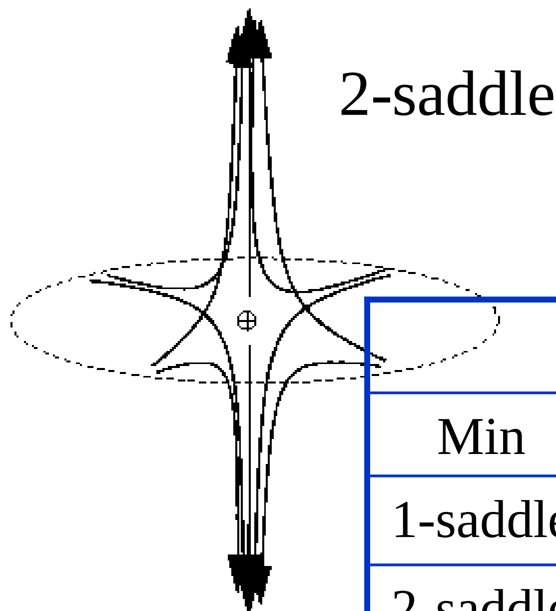


Ascending/Descending Manifolds

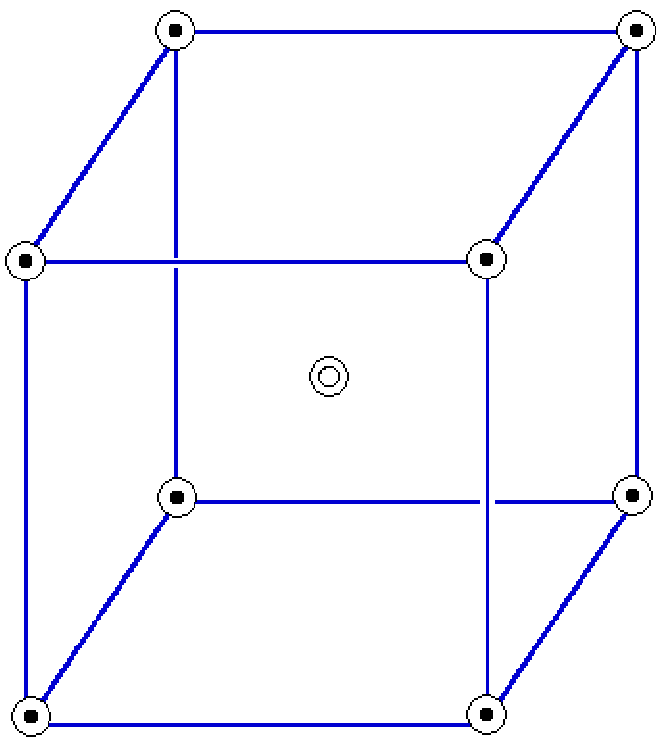
Ascending manifold: Points with common origin.

Descending manifold: Points with common destination.



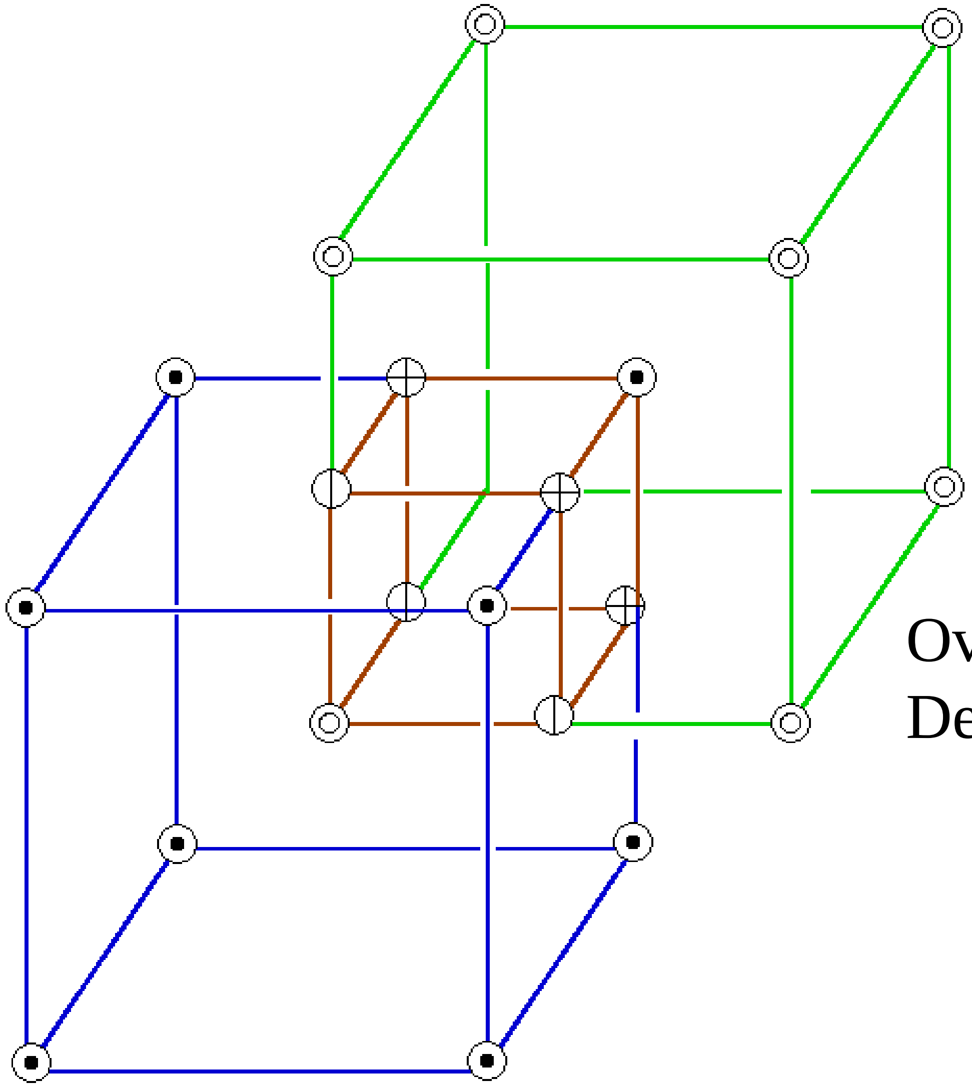


	Desc mfld	Asc mfld
Min	0-cell	3-cell
1-saddle	1-cell	2-cell
2-saddle	2-cell	1-cell
Max	3-cell	0-cell



Ascending manifolds
partition space

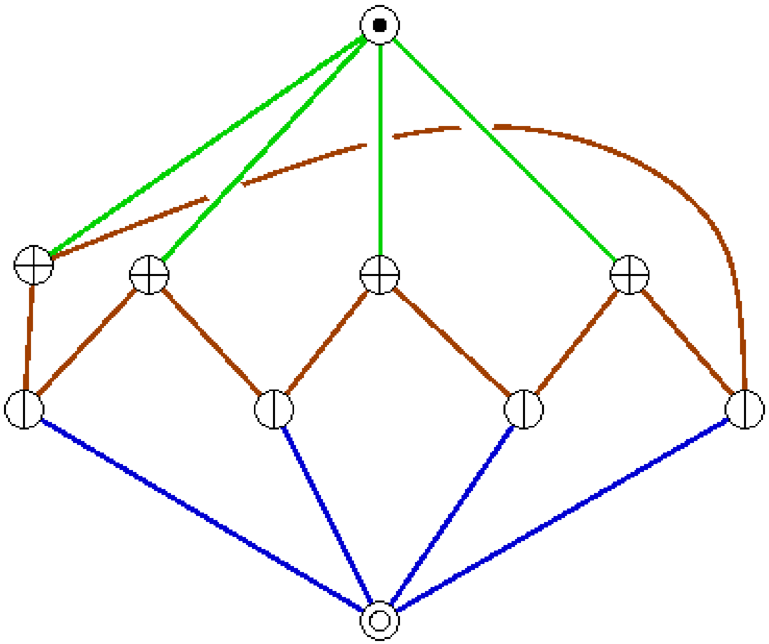
Morse-Smale Complex



Overlay of Asc and Desc manifolds

Morse-Smale Complex

A *cell* is a connected component of points with common origin and destination



Node

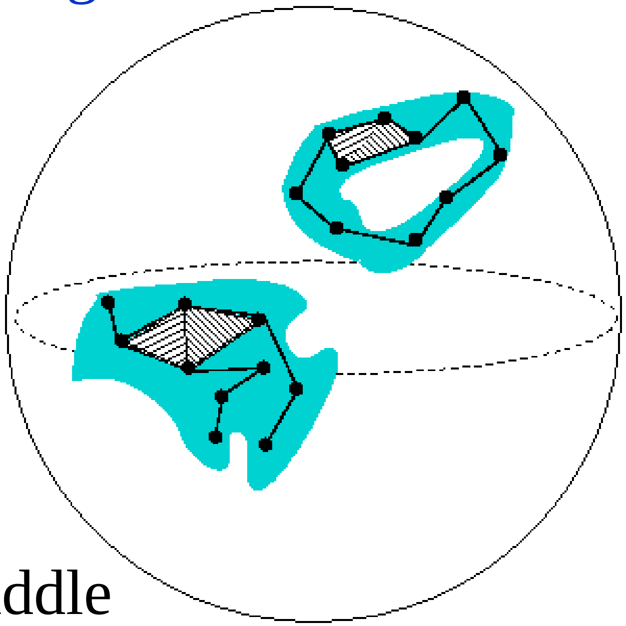
Arc

Quadrangle

Cyrstal

Continuous to Piecewise Linear

- Input: tetrahedral mesh, density at vertices
- Quasi Morse-Smale complexes
  - same combinatorial property
  - cells monotonic and non-crossing
- Critical points characterized by lower link



1-saddle + 2-saddle

Construction

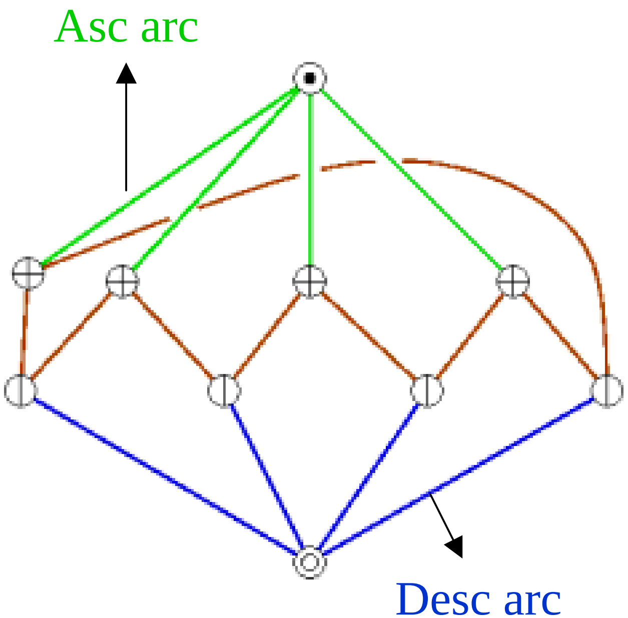
0. Sort Vertices
1. Downward sweep

   descending 1- and 2-manifolds
2. Upward sweep

   ascending 1- and 2-manifolds

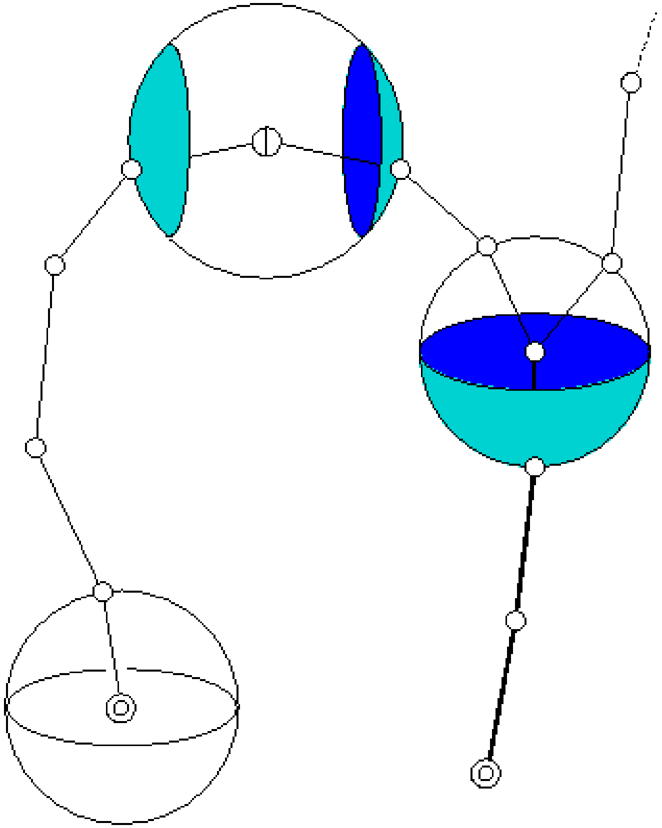
   + intersection curves

   using descending structure

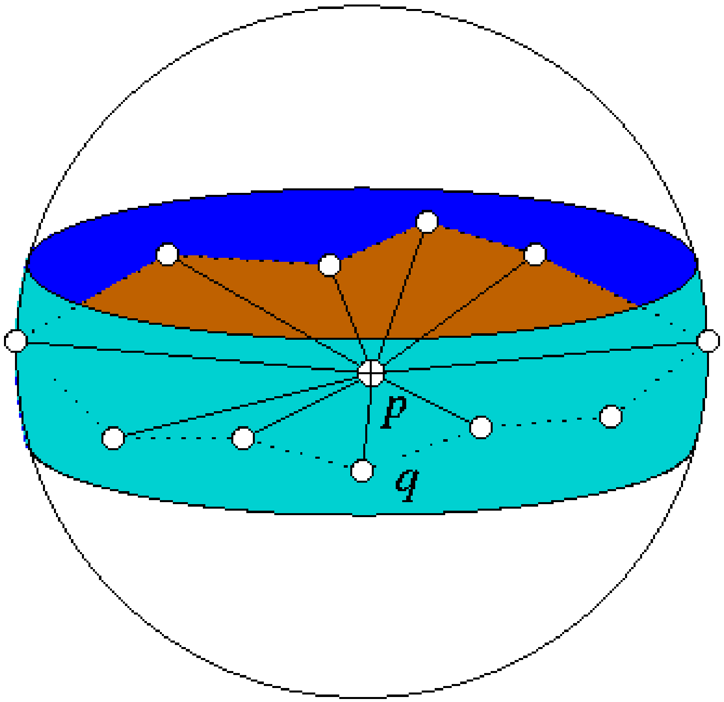


High Level Operations

- Starting (at 1-saddles)
- Extending (at all vertices)
- Gluing (at minima)



Desc arc construction



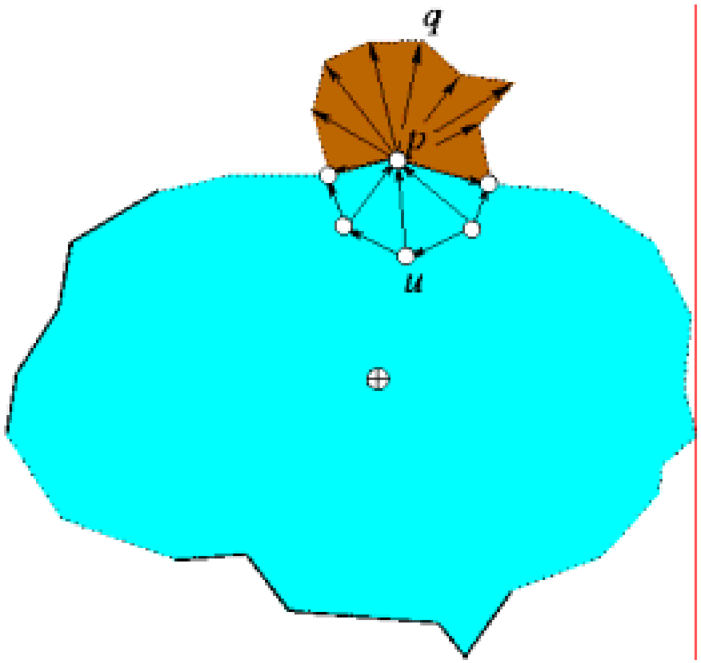
START DISK

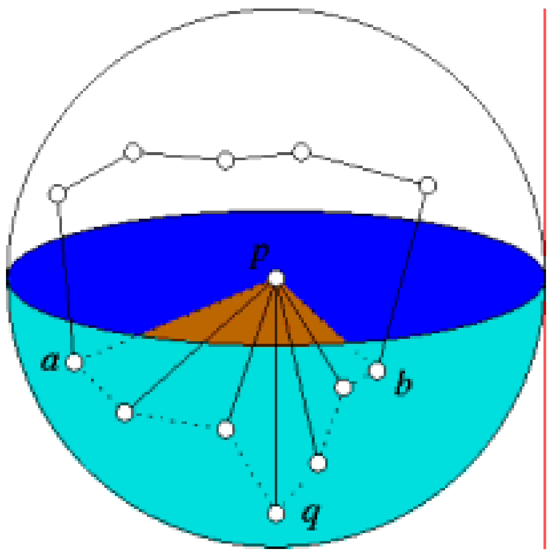
i. Construct shortest path tree of ocean rooted at lowest vertex *q*

ii. Discard non-tree edges by repeated edge-triangle collapse

iii. Choose edge that minimizes cycle length

iv. Add triangles from *p* to fill disk





Disk Invariant: Interior edges always directed towards unfrozen boundary vertices

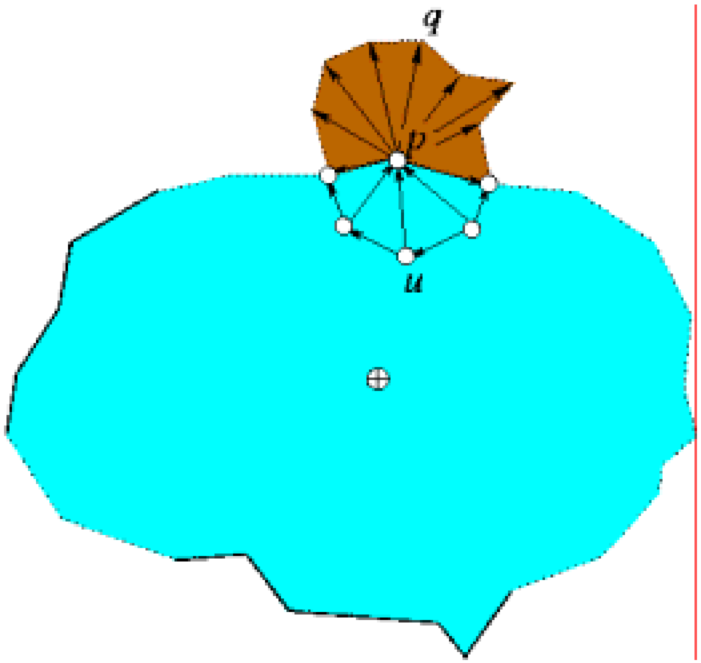
⇒ no interior edges between unfrozen boundary vertices

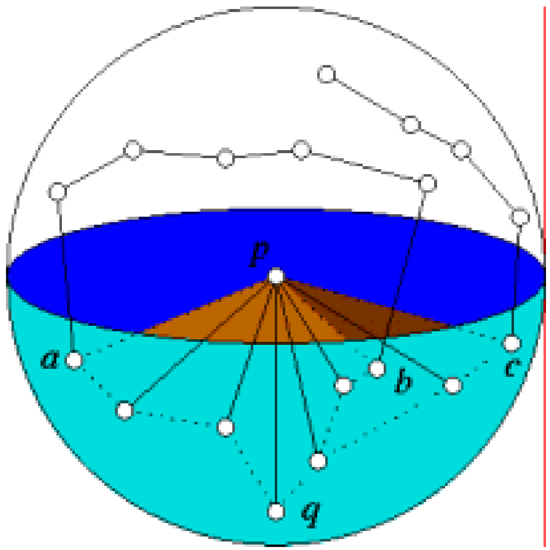
EXTEND DISK

i. Compute shortest path tree rooted at q

ii. Trace paths from a and b to q

iii. Add triangles to extend disk





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EXTEND DISK

i. Compute shortest path tree rooted at q

ii. Trace paths from a and b to q

iii. Add triangles to extend disk

Simultaneous Construction

At p :

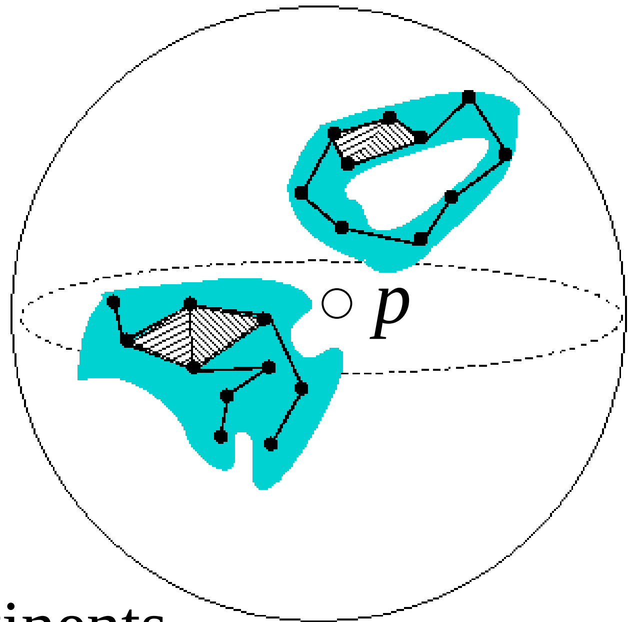
1.1. Start β_1 desc disks

1.2. Prepare $(\beta_0 - 1)$ asc disks

1.3. Extend desc disks touching p

1.4. Start $(\beta_0 - 1)$ desc arcs

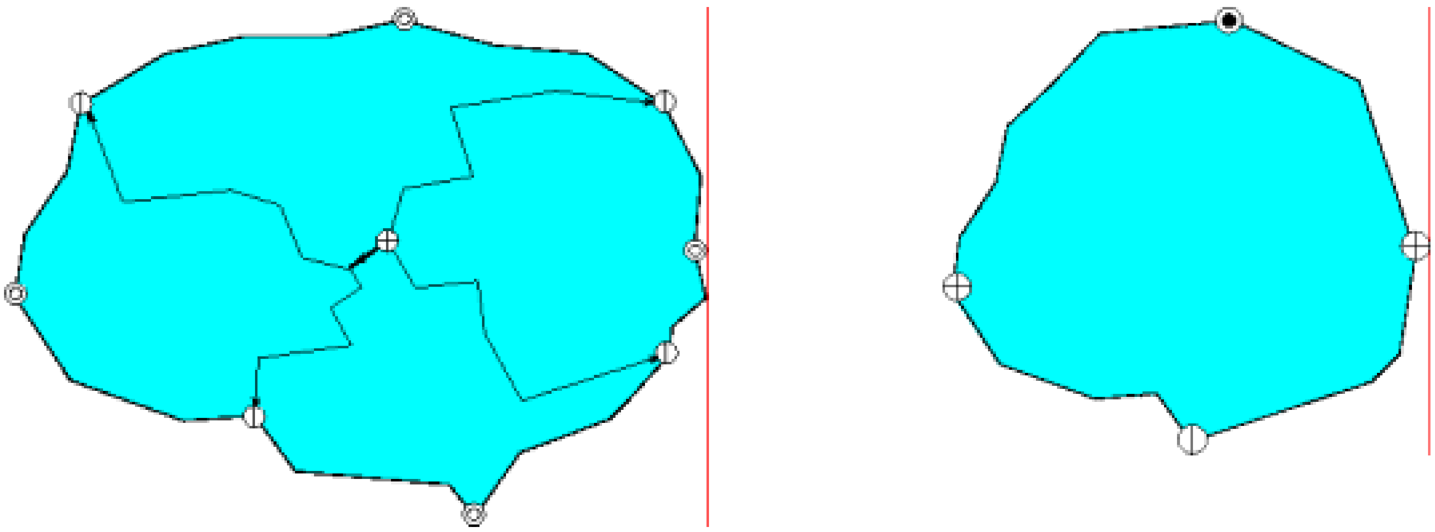
1.5. Extend desc arcs touching p



$\beta_0 = 2$ oceans

$\beta_1 + 1 = 2$ continents

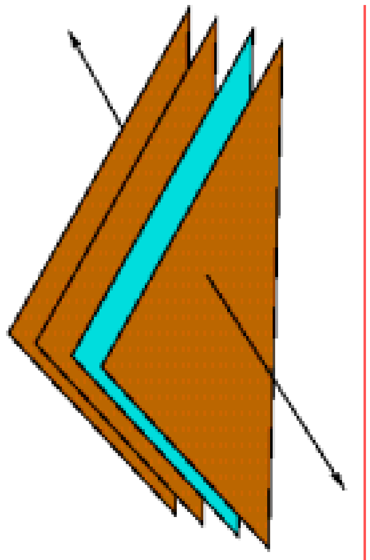
Ascending Manifolds

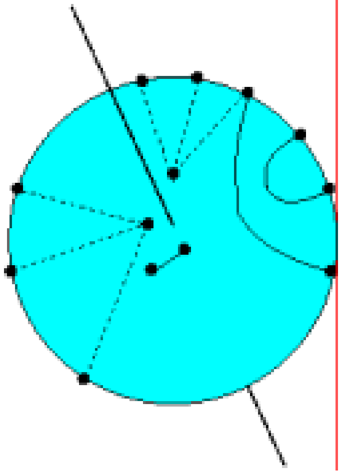


- Intersection curves
- Ascending arcs
- Ascending quadrangles

Simulating Disjointness

- Normal interval
  - order disks passing through a triangle
- Normal disk
  - order disks and arcs passing through an edge





Future Work

- Quasi to final complex
  - generalize handle slides from [EHZ 2001]
- Hierarchy
  - cancel pairs of critical points ordered by persistence [ELZ 2000] [EHZ 2001]
- Implementation
- Applications

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